Version of February 11, 2006

Speaking of Graphics

Chapter 2

Mercator, Wright and Mapmaking

2.1 Geographic maps as bivariate graphics

In the chapter on quipu we have described the univariate nature of the accounting system of centralized Inca government. Univariate means that a collection of objects (or subjects) is described from a single point of view: so much of this, so much of that, and so on. Because of their lack of written and printed communication, the Incas were not confronted with the two-dimensional space of tablets, parchment and paper. The latter offer the possibility for a bivariate graphic, in which some part of the world is viewed simultaneously from two points of view. It appears that the bivariate graphic display has been independently developed in Europe and in Asia. The most obvious one is the geographic map in which points on the earth are represented by two of their geographical properties, namely longitude and latitude as measured on the globe. Today, we may regard the problem of mapping the earth as rather trivial, and we easily forget the long and arduous way which has led to modern cartography. We often disregard the inherent problems of mapping a three-dimensional object such as a sphere onto a plane. We also tend to blind ourselves to the inherent bias and subjectivity that is unavoidable in any visual display of our experience of a multidimensional reality. For this reason it seems worthwhile to investigate briefly the early history of mapmaking. The contributions of Gerard Mercator and Edward Wright will be discussed more extensively, because they discovered and applied the mathematics of projecting the globe into a plane.

Geographic maps have led to the development of functional graphics and of statistical charts. Functional graphics describe a property of an object (or subject) as a theoretical function of another property, such as the change of velocity of an object with respect to time, the evolution of the composition of a chemical mixture as a function of temperature, and so on. Statistical charts relate two properties that have been measured (or observed) in a set of objects (or subjects), such as mortality from heart disease and consumption of fat in various countries. This may answer questions about the relationship between mortality and fat consumption in the general population. The distinction between functional graphics and statistical charts is slight, however. Both require a degree of abstraction which leads to the representation of non-geometric properties (such as velocity, time, temperature, concentration, mortality, consumption, etc.) in a geometrical way. The transition from cartography to more general graphics and charts has also been long and tortuous. Eventually it led to the great synthesis between algebra and geometry, known as coordinate geometry, which was proposed by René Descartes around 1637, and which is discussed in a following chapter. In the meantime we sketch summarily the main events and concepts that have paved the way for the bivariate diagram.

2.2 Ancient Mapmaking

The starting point of our brief journey to the world of mapmaking lies in the 6th century BC in Greece, when Pythagoras and the Pythagorean School after him realized that the earth was a spherical object, although they had no idea of its size. In the 4th century BC, Aristotle defined six climatic parallels on the terrestrial globe (‘climata’) and introduced the notion of latitude as a property of places on earth. In early days, degrees of latitude were determined from the shadow cast by the sun at its highest point in the sky from a ‘gnomon’, basically a stick placed vertically in the soil. Later on, in the 3rd century BC, the Greek astronomer and geographer Eratosthenes, who worked at the library of Alexandria in Egypt, defined longitudes through well-known places such as Alexandria and Rhodes. In that time, longitudes were estimated from travel reports of voyagers and soldiers, especially from the campaigns of Alexander the Great. Eratosthenes also arrived at a remarkably accurate estimate of the perimeter of the earth. The latter was deduced from the difference of the heights of the sun at the summer solstititium in Alexandria and in Syene (today called Aswan). These two cities are situated approximately on the same meridian and their distance was estimated at 800 km. The difference of the heights of the sun at noon was determined as 1/50th of a full circle. Hence, the perimeter of the earth turned out to be 800 x 50 or 40,000 km. Although Eratosthenes' observations were rather crude, he had the good fortune that his errors cancelled out [1]. Once the length of the equator was known, one could convert distances along parallels into degrees of longitude by means of trigonometry. In the course of the first century BC, the Greek stoic philosopher Posidonius repeated Eratosthenes' observations, but using this time the declination of the bright star Canopus when it was just above the horizon. Unfortunately, this time, measurement errors did not cancel out and Posidonius' estimate of the length of the equator turned out to be 29,000 km, some 11,000 km shorter than that of Eratosthenes [2]. Subsequent scholars, among which Marinus of Thyrus, adopted the new estimate by Posidonius and, as we will see, the error survived until 1492 when Columbus set sail on a presumably shorter western route to China and India. Note that the use of a rectangular grid defined by degrees of longitudes and degrees of latitudes was already practiced by geographers in Posidonius' time.

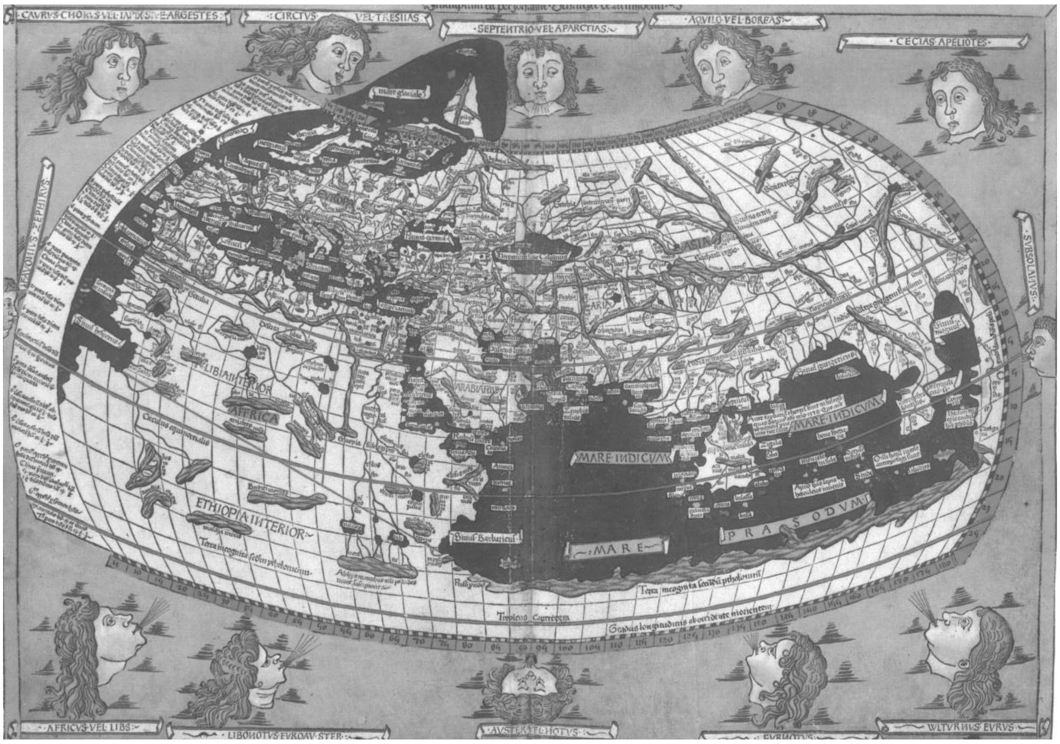


Figure 2.1. Woodcut reproduction of Ptolemaeus's map of the world by Nicolaus Germanus with curved meridians and parallels, from a German printed edition of the Cosmographia, which appeared in Ulm around 1482. The characteristic features of the Ptolemaic maps are the landlocked Indian Ocean, the absence of the Indian peninsula, the large island Taprobana, probably representing Ceylon, and the elongated shape of the Mediterranean Sea and of Scotland [3].

The most famous of the early Greek geographers and astronomers was Claudius Ptolemaeus (Ptolemy), who worked at the library of Alexandria, probably from 125-150 AD. Ptolemaeus is best known for his geocentric, more precisely geostatic, vision of the universe, which he described in his Megale Synthesis (Great Synthesis) or Almagest (from the Arabic Al Madjisti). His vision became the established doctrine well until the 16th century, when it was gradually replaced by Copernicus' heliocentric model. The Cosmographia (also referred to as Geographia) of Ptolemaeus contained a catalog of 8,000 places, of which some 400 were provided with (gnomonic) latitudes and with longitudes that were computed from traveler accounts. It also contained 27 maps, including one world map and 26 local ones. Ptolemaeus projected his world upon a cone which touched the earth globe at the 63rd parallel. The latter ran through the legendary island of Thule, the most northern part of the then known world (Ultima Thule has been identified with the Shetland Islands, the west coast of Norway and Iceland). He placed the zero meridian through the Fortunate islands (now called Canaries). This way, Ptolemaeus defined a conical grid (or graticule) which corrected for the shortening of the parallels at greater latitudes. In a first version, the parallels were drawn as curved lines, while the meridians are represented as straight lines. In a later version, both the parallels and the meridians appear to be curved (Fig. 2.1). Ptolemaeus's Cosmographia is the earliest attempt to separate facts from fiction, to present a coherent view of the world and to deal with the two-dimensional constraint of the plane. The Cosmographia certainly can be regarded as a milestone in mapmaking and has prepared the way toward more general coordinate-based charts. It provided the theoretical framework from which modern maps have been derived.

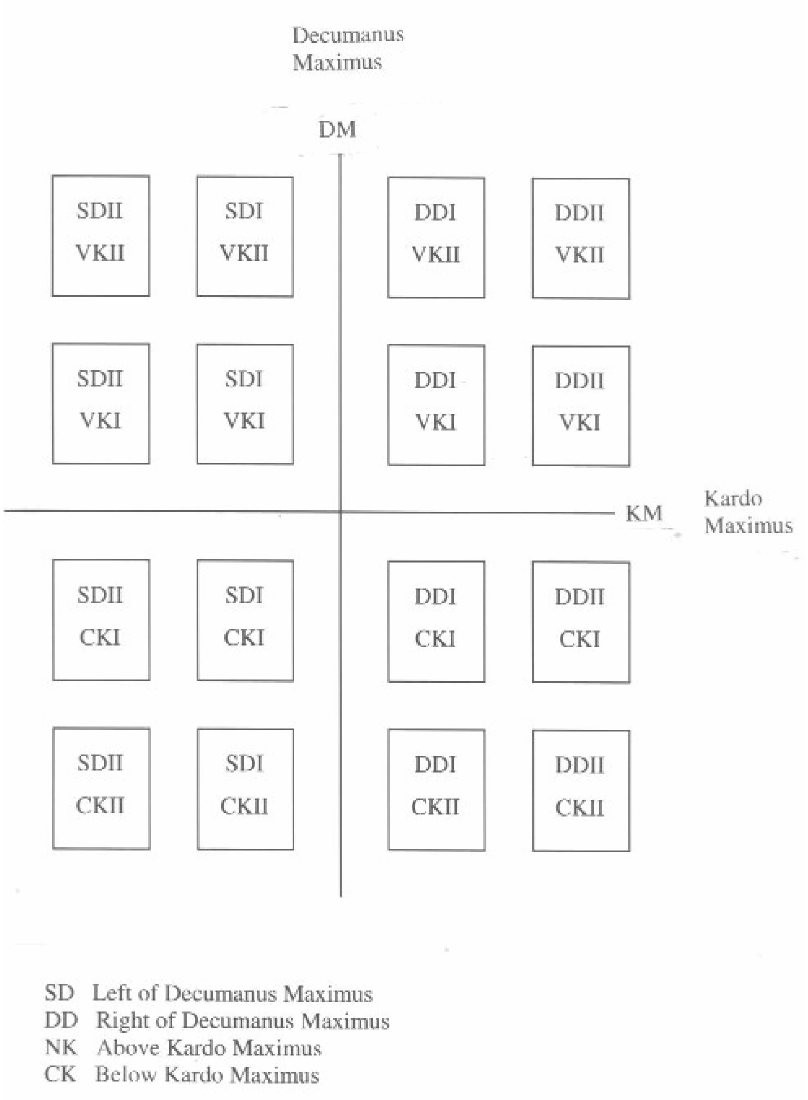


Figure 2.2. Coordinate system used by the Romans for cadastral mapping, called centuriation. Plots of land are defined by their position with respect to the Decumanus Maximus (DM) and Kardo Maximus (KM). Adapted from A. Hodgkiss [3].

In the Roman area, which extended into the 5th century AD, the great tradition initiated by Ptolemaeus came to a halt in Western Europe. It was only to be revived, as we will see, during the 15th century Renaissance, when the Cosmographia was rediscovered in Byzantium and brought to Italy. The Romans devoted more of their attention to practical matters. Their requirements for traveling were largely covered by so-called itineraries, which bore no relation with the actual geographic positions and directions. The Romans, however, invented a system of rectangular coordinates for the purpose of cadastral mapping and surveying, which was called ‘centuriation’. (Centuria is the Latin term for something that can be divided into hundred parts.) Its purpose was to divide newly won territories in rectangular plots for the purpose of colonization. Surveyors (agrimensors) made this division relative to a horizontal orientation called ‘Kardo’ (which means pen or fulcrum) and a vertical direction which was referred to as ‘Decumanus’ (which means line of division) [3]. The Roman grid system is illustrated in Fig. 2.2. Each plot is identified by its horizontal and vertical coordinates (Kardines and Decumani). For example, the notation SDII and VKI means two positions at the left (S) of the Decumanus Maximus (D) and at the same time one (I) position above (V) the Kardo Maximus (K). In this case the coordinate system is used as a two-dimensional index for retrieving information on a map. It is perhaps one of the oldest examples of a bivariate display of information on a rectangular grid. Synchronously with the late Roman period, the rectangular grid appears to have been in use in China since the 1st century AD. The astronomer Chang Heng, who worked in that period, 'cast a network about heaven and earth, and reckoned on the basis of it' [4]. The construction of a rectangular grid forms one of the Six Mapmaking Rules formulated by Phei-Hsui (or PeiXui) in 267 AD [5].

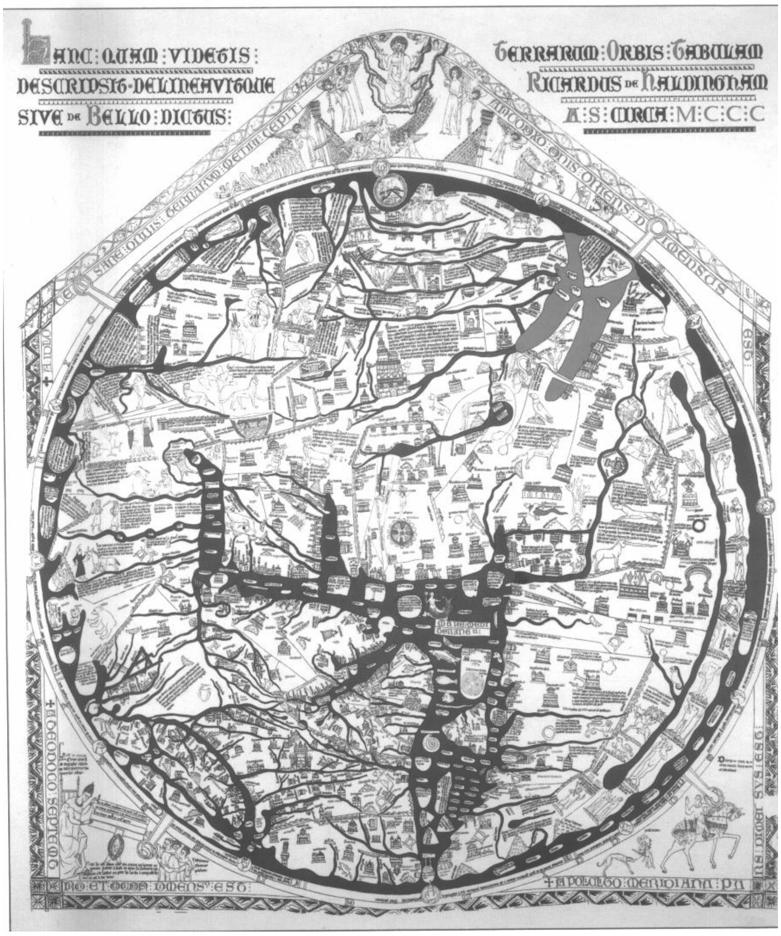


Figure 2.3. TO map of around 1300, known as the Hereford Mappa Mundi. East is on top and points towards paradise. Jerusalem is in the middle. Asia is separated from Europe and Africa by the horizontal bar representing the Don and the Nile. The vertical bar corresponds with the Mediterranean Sea and separates Europe (bottom left) from Africa (bottom right). (From Wychwood Antique Reproduction Maps.)

During the Middle Ages, Western Europe produced the so-called TO maps. The designation TO (also T-O or T in O) refers to the circular conception of the medieval world (orbis terrarum) and to its characteristic division into Europe, Africa and Asia. The Don and the Nile form the horizontal bar of the T , separating Asia (in the upper part) from Europe (at the lower left) and Africa (at the lower right). The latter two are separated by the Mediterranean Sea which forms the vertical bar, and the whole is surrounded by ocean (Fig. 2.3). The TO map presents a biblical view of the world, with Jerusalem at the center and the paradise often located on top and pointing toward the east. It also reflects the medieval division of the world between Noah's three sons (Jafeth, Cham and Shem who are associated with Europe, Africa and Asia, respectively). These maps were vehicles for the dissemination of theological ideas, rather than the products of scientific investigation. Within the context of scholastic teaching they served the goal of establishing doctrine and ensuring unity of thought. In contrast to their shortcomings from a rational point of view, they illustrate the power of visual displays, in particular of charts and graphics.

2.3 Early European mapmaking

Between the end of the Middle Ages and the beginning of the Renaissance, Nicole Oresme (1320-1382) applied the concept of longitude and latitude to physical properties of objects, other than geographical or geometrical. This French natural philosopher described the movement of an object in terms of longitude (time) and latitude (velocity) [6]. He also seems to have understood the multidimensional aspect of abstract space, for example by considering different parts of an object, each endowed with a different set of latitudes and thus generating three- or even higher-dimensional spaces. Oresme appears to be the first to have associated geometrical space with abstract physical properties such as time, velocity, temperature, and so on. We will discuss the contribution of Oresme in greater depth in the section on Descartes and coordinate geometry. In this overview of the contribution of ancient cartography to modern graphics, we only emphasize how the concept of purely geometrical space of longitudes and latitudes acquired a more abstract character. The resulting shapes of lines and surfaces in abstract spaces were the subject of intense discussion in the 14th century. These were called 'figuratio' or 'configuratio' and were thought to represent concrete qualities of the objects to which they applied. For example, the shape of the surface formed by the various latitudes of temperature in different points of a surface was thought to be responsible for how the object felt or tasted. Oresme also discovered that the area under the velocity-time diagram represented the distance traveled by the object. In this respect, he advanced on the results of integral calculus by some 350 years [7]. On the one hand, geometrical space became more abstract, as a result of uncoupling of longitude and latitude from the obvious geometrical dimensions of length. On the other hand, the curves and surfaces generated within such abstract spaces were assigned a high degree of reality within the realm of human experience. The different interpretations of graphics as either conventional descriptions without any further significance, or as revelations of a hidden reality are still of actuality. In a broad sense, the scholastic battle between nominalists and universalists still rages on in our time, as it is the case in quantum physics.

Figure 2.4. One of the 27 local maps from Ptolemaeus's Cosmographia representing Gallia (including the Netherlands, Belgium and France) from an early Italian illuminated edition of 1406. From L. Pagali [2].

Navigational instruments were improved considerably during the 12th and 13th century. This included the introduction of the compass and of various instruments for measuring the height of the sun and the declinations of stars, such as the astrolabe, the quadrant and the cross-staff (which were rudimentary precursors of the modern sextant). In those times, sailors used 'portolan' maps which indicated harbors and outlined the shores of their destinations. Portolan maps were only useful for sailing close to shore. In the open seas, gross errors occurred when charting a straight compass bearing (or loxodrome) especially at greater latitudes. This problem was only to be solved in the 16th century. The great turning point in scientific cartography arrived during the early Renaissance, when scholars from Byzantium visited Italy and brought with them copies of Greek manuscripts from the library of Alexandria. Among those was Manuel Chrisoloras who moved to Florence in 1397 and took with him a copy in Greek of Ptolemaeus's Cosmographia. His student Jacopo Angelo translated the Greek text into Latin in 1406, and changed its title into Cosmographia, which was more to the taste of the Renaissance humanists [2]. The 1406 edition of the Cosmographia also provided a listing of each place referring to the local map in which it appeared together with its longitude and latitude (Fig. 2.4). This way, coordinates were used as two-dimensional indices for retrieving places on maps. It exemplifies the relation between tabulated numbers and geometric positions in space.

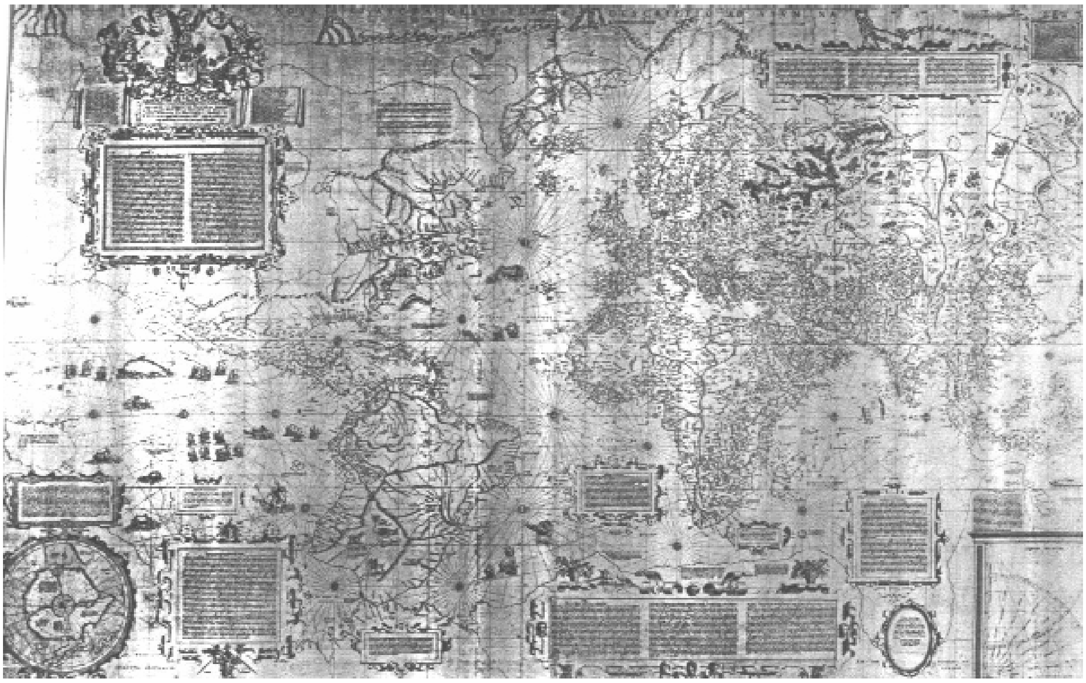


Figure 2.5. Mercator's world map of 1569 with a rectangular grid in which distances between parallels increase toward the poles [10].

A decisive milestone in cartography was the invention in Europe of the printing press, presumably by Gutenberg around 1440. The collateral effect of this technological advance in mass reproduction was the development of woodcutting, mostly in Germany, and copper plate engraving in Italy. In 1492, Columbus sailed with three ships along a presumably shorter western route to China and India. On board he may have carried a copy of the Cosmographia, printed in 1474 by Johann Müller (Regiomontanus). This world map still contained Ptolemaeus's erroneous length of the equator, which was 11,000 km less than in reality and, as it turned out, a most fortunate mistake. The discovery of the new continent brought enormous riches to Spain and increased the need for new maps and more accurate navigation. From 1524, the Cosmographia has been extended in numerous German printed editions by Sebastian Müller and Peter Bienewitz (Apianus). In that period Gemma Frisius popularized the method of triangulation, by means of which land surveys could be made more accurately than before. Around 1550 Flemish copper plate engravers reached a high standard of excellence and became leading cartographers during the second half of the 16th century. The better known of them include Gerard Mercator whose 'Atlas sive Cosmographicae Meditationes' was published posthumously in 1595, Abraham Ortelius whose 'Theatrum Orbis Terrarum' appeared in 1570, Gerardus de Jode with his 'Speculum Orbis Terrarum' of 1578 and Jodocus Hondius (Josse de Hond) who produced a new and enlarged edition of Mercator's Atlas in 1606. Mercator is known especially for his application of a scale of increasing latitudes in his world map of 1569 'ad usum navigantium' on which courses with constant compass bearing could be plotted as straight lines (Fig. 2.5).

Mercator never disclosed the method that he employed for the construction of his scale of increasing latitudes, although it is speculated that he used a geometrical approach rather than calculus. The mathematical form of the Mercator projection was discovered afterwards by Edward Wright and was published in 1599 in his famous book entitled 'Certaine Errors of Navigation' [8]. The invention of the geographical projection marks a turning point in chartmaking, as never before mathematical techniques were applied with such accuracy. From that moment science started to take over from art, and emphasis shifted from the propagation of ideas to the recording of facts. For these reasons, we will expand somewhat longer on the life and work of Mercator and Wright later in this chapter. With the flight of Mercator and Hondius from religious intolerance in Flanders, mapmaking moved to the North and was continued by Dutch engravers. Among the most famous of them are Willem Blaeu and his son Jan who compiled a new 'Theatrum Orbis Terrarum' in 1635. The new maps discontinued the ancient Ptolemaic tradition, as they were based on the most recent observations by voyagers and geographers. At the end of the 17th century, latitudes could be determined rather accurately from observations of the height of the sun or from the declination of the stars. By means of the sextant devised by Isaac Newton, latitudes were determined within 10 angular minutes, depending upon weather conditions. (One angular minute is the equivalent of roughly one mile.) The determination of longitude remained a problem, however. At first, this was obtained by means of the pendular clock, devised by Christian Huyghens (1656), and later from tables produced by Jean Dominique Cassini (1669) for the eclipses of the moons of Jupiter. In 1707 the English Board of Longitude was installed after four ships of the Royal Navy wrecked on the Scilly Islands because of navigational error. A prize of 20,000 pounds (about 1 million pounds in today's money) was awarded for a practical and reliable method of determination of longitude aboard ships. John Harrison's chronometer eventually won the prize in 1736, when his instrument kept time within 15 seconds during the 165 days of Captain Cook's world voyage. In the 18th century, mapmaking became progressively dominated by French cartographers and gradually turned into a science of its own. In our times, cartography and navigation has reached an unprecedented degree of accuracy due to satellite surveys. Nowadays, positions can be determined within a few meters, depending upon the number of satellites sighted and the distances between them.

2.4 Flattening the sphere

From ancient times, a dilemma has existed between the objectivity of observations and the subjectivity of their graphic representations. This is the reason why we insist on geographical cartography as a specialized paradigm of statistical chart making. In designing charts, one has to make choices which depend upon the subject's point of view and vision. No single chart, whether geographic map or statistical chart, can claim to represent objectively the world or even a major part of it. In particular, when projecting the globe on a plane surface it is not possible to fully satisfy more than one of the following three criteria: conformity of angles, equidistance and equivalence of areas. The Mercator projection conforms with respect to angles since constant compass bearings are represented by straight lines (loxodromes or rhumb lines), but shortest courses appear as circular paths (orthodromes) on it. The gnomonic or polar projection preserves equidistance and, hence, allows determining shortest distances, but it is not designed to chart constant compass bearings.

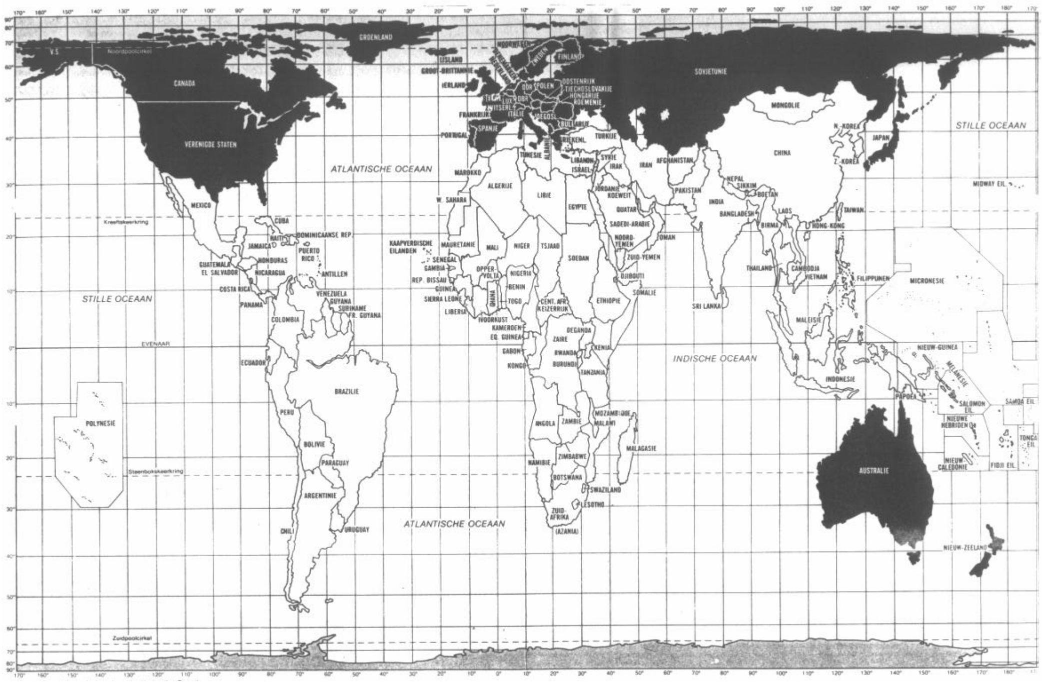


Figure 2.6. World map in the Arno Peters projection with equivalent areas [9]. In this equal area projection, distances between latitudes decrease in the direction of the poles, as opposed to the Mercator projection.

The Arno Peters map (1974) of the world shows the 'true' relation of the continents with respect to their area, but does not generally preserve angular and linear distances [9]. The Peters projection uses a rectangular grid with decreasing latitude, the opposite of the Mercator projection (Fig.2.6). It is recommended for education and for use by the mass media, especially TV, as it shows Africa, South America and South Asia in its true proportions with respect to Europe, North America and the rest of the world in terms of areas. Note that a deliberate choice is made here to propagate a social and political vision of the world and to satisfy the need and fashion of our time. This is what Mercator and the geographers of the 16th century also did in an exemplary way. For this reason we will retrace the discovery and the formalization of the Mercator projection in greater detail.

2.5 Gerard Mercator

Mercator is the Latinized name of Gerard De Cremer (literally the one who sells in the streets and on the markets) who was born in 1512 in Rupelmonde, a town in the then Southern Netherlands. In 1532 he became Magister Artium at the University of Leuven where he studied under Gemma Frisius (Jemme Reinesz, originating from Friesland), professor of medicine and mathematics and developer of the method of triangulation. Frisius also wrote numerous textbooks on arithmetic and operated a workshop for the construction of surveying instruments and globes. After his graduation Mercator spent a few years in Antwerp in the (vain) hope of finding rich sponsors for his scientific projects. During this period, Mercator learned calligraphy, engraving and the art of instrument making from reputed craftsmen. In 1537 he opened his own workshop in Leuven for the manufacture of instruments, globes and maps. In the same year, he produced a map of Palestine (Terra Sancta) for the use by Christian pilgrims to the holy places. A map of Flanders based on triangulations by Gemma Frisius was published in 1540. The following year (1541) Mercator offered an earth globe, with loxodromes drawn on it, to Nicolas de Granvelle, chancellor of Emperor Charles V . These loxodromes on the globe appear as straight lines that intersect with the meridians at constant angles. It is likely that Mercator's earth globe was designed in view of solving problems in navigation. He seems to have been acquainted with nautical problems through his contacts with renowned Portuguese and English navigators such as Pedro Nuñez and John Dee. Mercator's work was interrupted, however, in 1544 when he was accused of heresy (‘Lutheranism’) on the ground of certain 'suspect' letters that were never produced in evidence. After three or four months of incarceration in the castle of Rupelmonde, Mercator was released. In 1552 he left Flanders for Duisburg (in Germany, close to the Dutch border) and was appointed in 1566 as surveyor and cartographer of the duke of Gullik and Kleef (or Cleves). In this position he constructed his famous world map 'ad usum navigantium', using a cylindrical projection with increasing distances between parallels (Fig. 2.5). As will be explained below, the projection preserves angles in every direction. It satisfies one of the three criteria of geographical projection, but violates the other two (equidistance and equivalence of areas).

The idea of increasing latitudes in a cylindrical projection must have been circulating among cartographers of the 16th century. It is speculated that Mercator could have been inspired by a map engraved on the back of sundials which were manufactured by Erhardt Etzlaub (1511) for the use by pilgrims [10]. Mercator never formally described the transformation of the scale of latitudes on his famous world map of 1569. Possibly, Mercator wanted to protect the secrets of his craft. Although he had studied mathematics under the distinguished Gemma Frisius, his knowledge of mathematics may not have been beyond geometry and elementary trigonometry. For this reason it is maintained that his construction was geometrical and graphical rather than by calculus and by the use of tabulated trigonometric functions [11].

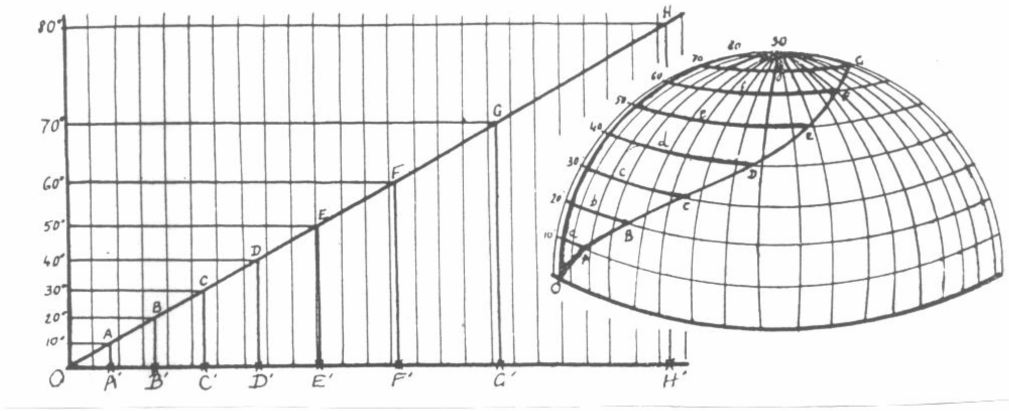


Figure 2.7. Geometrical construction which may have been used by Mercator for the construction of his scale of increasing distances between parallels in his world map of 1569 [12].

A speculative construction is shown in Fig. 2.7 which has been reproduced from H. Nauts [12]. In this figure, a loxodrome is constructed on a globe which intersects the meridians at a constant angle of 60 degrees. At equal intervals along a chosen meridian, parallel line segments a, b, c, d, ... are drawn which meet the loxodrome at the points A, B, C, D, ... These segments are reported along the horizontal axis of a rectangular grid and define the points A', B', C', D', ... Perpendicular line segments through these points intersect with the straight line through the origin and at an angle of 60 degrees with the vertical axis. These intersections are the images of the points A, B, C, D, ... on the globe. They define the divisions of the scale of increasing latitudes in the rectangular grid. Measurements performed on a copy of the 1569 world map of Mercator show rather important deviations from the theoretical values [12].

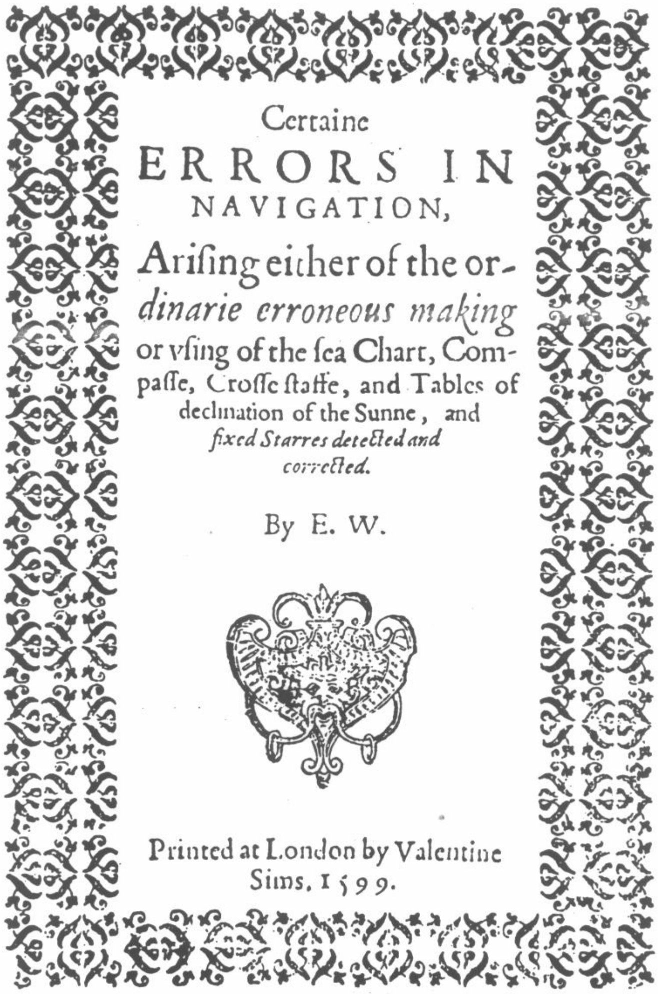
φ deg.	y (Mercator) cm	y $\left(\ln\frac{\cos\varphi}{1-\sin\varphi}\right)$ cm	Relative Difference %
10	5.45	5.43	0.37
20	11.11	11.15	-0.36
30	17.01	17.00	0.06
40	23.54	24.80	-5.08
50	31.02	31.27	-0.80
60	40.24	40.75	-1.25
70	52.86	53.69	-1.55

Table 2.1. Theoretical distances from the equator at different latitudes (φ) in comparison with those measured on Mercator's world map of 1569 by Emm. de Martonne in 1948 [12]. The theoretical distances have been scaled such as to be comparable with the distances (in cm) on the Mercator map of 1569.

Table 1 shows the computed and measured distances of the parallels from the equator at various degrees of latitude. (The derivation of the formula will be given later on.) The agreement is fair for the smaller latitudes up to 30 degrees, but discrepancies vary rather non-systematically, at higher latitudes. (Note the anomaly at 40 degrees.) Various theories, mostly geometrical, have been proposed in order to reconstruct the process by which Mercator may have arrived at his projection. A problem is that measurements on the original chart cannot be relied upon, due to anisotropic shrinkage of the paper which may have resulted in the course of 400 years. Since the original copper engraving has been lost, the enigma around the Mercator projection may remain unresolved for ever.

After completion of the world map, Mercator prepared an edition of Ptolemaeus' Cosmographia (1585), followed by several series of maps of France, Switzerland, Germany, the Netherlands, Italy and the Balkan countries (1589). Gerard Mercator died in 1594 at Duisburg at the age of 82 years. In 1595, one year after Mercator's death, his son Rumoldus published the famous Mercator Atlas. The front page shows Atlas, a legendary king of Etruria, who was reputed for his wisdom, holding the earth globe on his knee. (Mercator's Atlas bears no relation to the Greek titan who was sentenced by Zeus to carry the earth on his shoulders as a punishment for storming the heavens.) Since then, the word atlas has acquired a more generic meaning for a collection of graphical displays, such as an anatomical atlas or the political and economical Atlas by William Playfair (1776), to which a chapter in this book is devoted. As we have already related above, Mercator's legacy was acquired by Jodocus Hondius (Josse de Hond) in 1602, who published the enlarged Mercator-Hondius Atlas. Hondius was a Flemish engraver who fled to London in 1585 when Gent was besieged by Spanish troops. In London he engraved for the ‘Mariners Mirrour’ of Lucas Jansz Waghenaer and for the records of the voyages of Francis Drake. He also became acquainted with the English mathematician Edward Wright to which we will refer again below. Upon his return in Amsterdam (in 1596 or 1597), he published a world map using the Mercator projection, a remarkable feat, since it was known that Mercator never had admitted of how the projection should be constructed and that he had taken the secret to his grave. At this point the history of the Mercator projection almost takes the form of a detective story, in which Jodocus Hondius and many others played a role around the central figure of Edward Wright, one of the leading mathematicians of Elizabethan England [13]. The story is worth to be related in detail.

2.6 Wright's form of the Mercator projection


Certaine
ERRORS IN
NAVIGATION,
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declination of the Sunne, and
fixed Starres detected and
corrected.

By E. W.

Printed at London by Valentine
Sims. 1599.

Figure 2.8. Title page of Edward Wright's 'Certaine Errors in Navigation' published in 1599 [8].

Edward Wright was born in 1561 at Garveston, near Norfolk, in a family with modest income (mediocris fortunae) [14]. In 1576 he entered Gonville and Caius College in Cambridge where he obtained a fellowship in 1587. Possibly, he has been absent from Cambridge during the period 1581-1584 and was employed at sea. In 1589 Wright obtained royal permission to join the expedition of Georges, Earl of Cumberland, to the Azores in order to prey upon Spanish ships and to obtain booty from them. He returned to Cambridge the same year. Three years later (1592) he presented his manuscript 'Certaine Errors in Navigation' to the Earl of Cumberland (Fig. 2.8). The work provided a mathematical justification of Mercator's projection, followed by a discussion of the variation of the compass and of the errors arising from the use of the cross staff and various astronomical tables for the determination of latitude. There is no doubt that Edward Wright had discovered the mathematical form of the projection which Mercator had first used in 1569 but was reluctant to reveal [15].

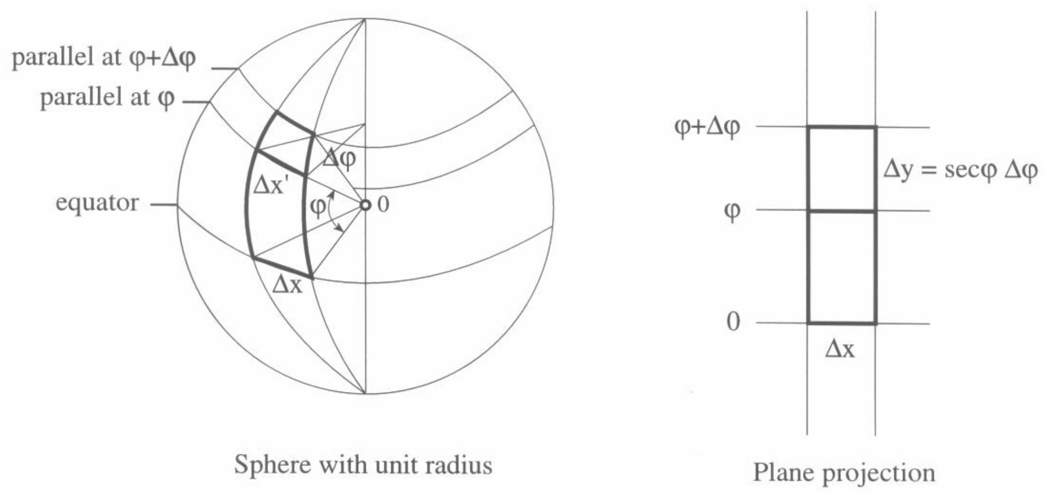

Figure 2.9. Geometrical interpretation of the Mercator-Wright projection. For explanation see text. The ratio of Δy to Δx on the map is the same as that of $\Delta\varphi$ to $\Delta x'$ on the sphere (assuming a sphere with unit radius).

The central idea of Wright's mathematical form is straightforward [16]: 'I first thought of correcting so many gross errors ... in the sea chart, by increasing the distances of the parallels, from the equinoctial towards the poles, in such sort, that at every point of latitude in the chart, a part of the meridian might have the same proportion to the like part of the parallel, that it has in the globe.' For the projection of the unit sphere on a plane according to Mercator and Wright, we would write in modern notation that:

$$\frac{\Delta y}{\Delta x} = \frac{\Delta \varphi}{\Delta x'} = \frac{\Delta \varphi}{\cos\varphi \, \Delta x}$$

where Δy represents the distance between parallels in the projection at a given latitude of φ degrees, where Δx is the distance between meridians at the equator and $\Delta x'$ is the corresponding distance on the sphere at the latitude of φ degrees (Fig. 2.9). The relation can be simplified such as to provide Δy as a function of φ for every φ :

$$\Delta y = \sec\varphi \, \Delta\varphi$$

In order to find the form of y as a function of φ one can solve the definite integral:

$$y = \int_0^{\varphi} \sec\varphi' \, d\varphi' = \ln \frac{\cos x}{1 - \sin x}$$

or equivalently:

$$y = \ln \tan\left(\frac{\varphi}{2} + \frac{\pi}{4}\right)$$

where the symbol ln means natural logarithm (with base $e = 2.71828...$). The logarithmic transformation function was discovered 30 years after Wright's death by Henry Bond in 1645 [16].

In Wright's time, integral calculus had not yet been invented, and even logarithms were not heard of prior to 1614. Instead Wright solved the integral by means of a finite sum over successive one-degree intervals of φ [17]: '...by perpetual addition of the secantes answerable to the latitudes of each point or parallel into the summe compounded of all the former secantes.' This approach would be used today on a digital computer as a (rough) method of integration using sufficiently small constant intervals $\Delta\varphi$:

$$y = \Delta\varphi \sum_{i=1}^{n} \sec \varphi_i \qquad \text{with } \varphi_i = i\,\Delta\varphi \text{ and } n = \varphi / \Delta\varphi$$

This way Wright constructed a table which allowed to convert latitudes (φ) into distances from the equator (y) using values of $\sec \varphi$ from a trigonometric table (Canon Triangularum) published by Georg Joachim Rheticus (1514-1574). In a later edition Wright refined his calculation using even smaller intervals [8]: 'A table is calculated and the use thereof shewed for the true and easie dividing of the meridians into tennes of minutes of degrees of latitude, proportionally increasing towards the poles'.

φ deg.	y (Wright)	y $\left(\ln \frac{\cos \varphi}{1-\sin \varphi}\right)$ x 34,373.468	Relative Difference %
0	0	0	.000
10	6,030	6,030	.000
20	12,251	12,250	.008
30	18,884	18,882	.011
40	26,228	26,224	.015
50	34,746	34,741	.014
60	45,277	45,268	.020
70	59,667	59,652	.025
80	83,773	83,742	.037
85	107,696	107,634	.058
89	163,173	162,977	.120

Table 2. 2. Comparison between the results of Wright's numerical integration (1599) and the exact value as computed from the transformation function which relates degrees of latitude to distances from the equator in the plane projection.

Table 2 shows the distances from the equator in the Mercator-Wright projection for some selected values of latitude as reported by Wright, together with the exact values derived from the solution of the integral and the corresponding relative error. In Wright's table the secant of 10 minutes is set to 100 units and the computed values were scaled accordingly for the purpose of comparison. Note that the agreement between the numerical approximation and the exact value is quite remarkable (except for latitudes close to 90 degrees). Using Wright's table one could now produce a map in the Mercator projection for every point on the globe for which the longitude and latitude had been determined.

From historical records appears that Mercator's secret has been broken even before his death in 1594. Wright was slow to publish his remarkable finding, but was rather generous in distributing copies of his manuscript. As a result, his friend, Thomas Blundeville published Wright's table of the subdivision of the meridian in his 'Exercises' of 1594, with due acknowledgement to the author. In the same year, Wright moved from Cambridge to London where he acquainted with Jodocus Hondius, who obtained a copy of 'Certaine Errors' under promise not to divulge the content. After his return to Amsterdam, however, despite his promise, Hondius published in 1596 (or 1597) his famous 'Christian Knight' world maps upon the projection formulated by Wright without giving credit, however, to him. Around that time (1597), William Barlow published Wright's table in 'The Navigator's Supply', with only a cryptic reference to its source. Richard Hakluyt, a renowned pilot and friend of Wright, published two world charts with the new projection in his 'Principal Navigations' of 1598. In the same year (or shortly thereafter) a manuscript was brought ashore that belonged to a navigator (presumably Abraham Kendall), who died during Francis Drake's last expedition, and which had been prepared for publication. The manuscript was referred for appraisal to Wright who quickly discovered that it was a copy word for word of his own book. Eventually Edward Wright decided to publish his work in 1599. A revised and extended edition appeared in 1610. Even then his humility showed on the title page, which mentioned only his initials: 'Certaine Errors in Navigation ... By E. W’. In his 'Praeface' he noted wryly about Mercator's secret method: 'But the way how this should be done, I learned neither of Mercator, nor of any man else. And in that point I wish I had been as wise as he in keeping it more charily to myself.' The 1599 edition also described Wright's previous voyage with his patron, the Earl of

Cumberland, to the Azores and contained a map of the Atlantic part of Europe according to the calculated Mercator-Wright projection (Fig. 2.10).

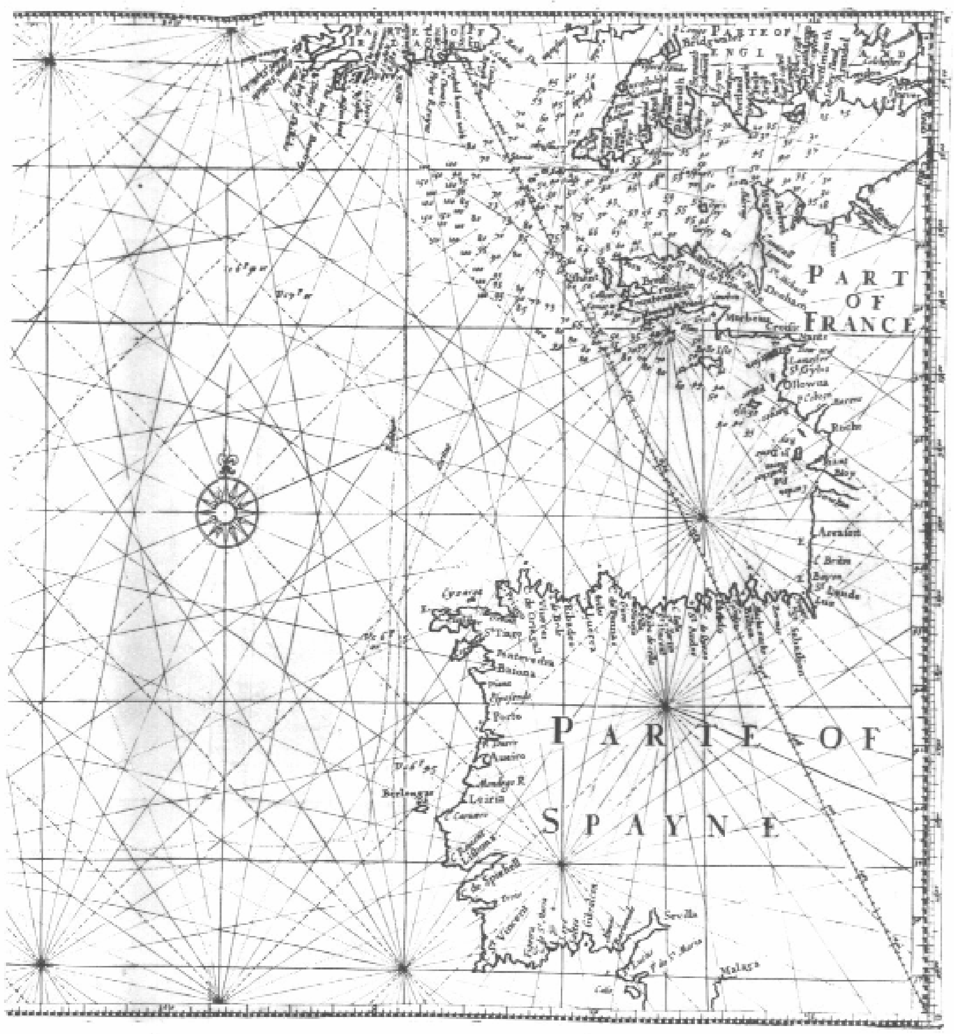

Figure 2.10. Edward Wright's map 'for sailing to the Isles of Azores' which was added to his 1599 edition of 'Certaine Errors in Navigation'. The map is constructed according to the Mercator-Wright projection which is described in the book. Note the star-like grids of constant compass bearings (loxodromes or rhumb lines) that are superimposed on the map [8].

Wright also appeared as a central figure in the mathematical world of his time. His efforts were primarily devoted to the improvement of navigational aids and in the promotion of new mathematical and physical discoveries rather than for his personal benefit. He translated Simon Stevin's 'Havenvinding' from Dutch into

English (The Haven-finding Art, 1599). He also assisted William Gilbert in the compilation of 'De Magnete' (1600) to which he contributed a preface. The latter book proposed a theory for the magnetism of the earth and discussed the causes of the variation of the compass. In 1608 (or 1609) Wright is appointed tutor to Prince Henry and in 1612 he is nominated lecturer in navigation in London with the support of the East India Company. He also acted as a surveyor for the New River project for bringing water to London. His last great work was the translation of Napier's work on logarithms 'Mirifici Logarithmorum' of 1614 from Latin into English [18]. This work was edited posthumously by Henry Briggs, who was a close friend. Edward Wright died in London in 1615 at the age of 54 years. In the English version of Napier's book Wright is acknowledged as 'That famous, learned, Errors true Corrector, England's great Pilot, Mariners Director' [19]. Nobody had done more to set the seal on the supremacy of England in the theory and the practice of the art of navigation in his time. An epitaph in the annals of Caius College reads: 'He studied more to serve the public than himself'.

Statistical chart making is closely related to geographical cartography. Both attempt to reproduce the world in which we live within the constrained space of a plane graphic, and, of necessity, present a distorted view of reality. Statistical charts as well as geographical maps emphasize certain properties of the world they describe, and at the same time omit many others. Commenting on the power of maps, Denis Wood stated that the usefulness of maps derives from their bias and subjectivity and that these are qualities to be highlighted and celebrated [20]. The same contradiction between utility and partiality applies equally, if not more so, to statistical graphics. The French philosopher of the sociology of sciences, Bruno Latour, regards graphics as inscriptions that form part of a vast social and cultural undertaking. In a sense they are vehicles by which ideas are advertised and marketed. They circulate freely among their audience while their message remains fixed and unchanged. As such, they are called 'immutable mobiles' by Latour [21]. It is no wonder that their number and use has increased as a result of the major innovations in visual mass communication, such as printing, television and electronic computing.

Notes

[1] Charles Bricker, Landmarks of Mapmaking. Sequoia, Lausanne, 1968. Eratosthenes assumed that Syene was exactly on the tropic of cancer, such that the height of the sun at the summer solstititium would be exactly 90 degrees there. In that case, he required only a gnomonic observation of the sun in Alexandria. In reality, Syene was 60 km north of the tropic. Furthermore, Alexandria was some 3 degrees east of Syene and their distance was overestimated by 80 km. Fortunately, however, the errors seem to have cancelled out, which led to the remarkably accurate estimate of the length of the equator at 40,000 km.

[2] Lelio Pagali, Claudii-Ptolemaei Cosmographia. Orsa Maggiore, Torriana, It., 1990. This book contains color reproductions of the 27 maps from the codex Lat. VF.32 in the National Library of Naples. It is one of the most beautifully written and illuminated copies of the Cosmographia, which first reappeared in Florence circa 1410.

[3] A.G. Hodgkiss, Understanding Maps, A systematic history of their use and development. Dawson, Folkestone, Engl., 1981. A profound, comprehensive and beautifully illustrated description of mapmaking, including statistical cartography, from the ancient times to modern developments.

[4] A.G. Hodgkiss, Understanding Maps. Opus cit.
The author also mentions an inscription in stone at Xian from 1137, which bears a rectangular graticule.

[5] Cao Wanru, Maps 2000 Years Ago and Ancient Cartographic Rules. In: Ancient China's Technology and Science Institute of the History of Natural Sciences. Foreign Language Press, Beijing, 1983, pp. 251-257.
The Six Mapmaking Rules refer to (1) graduated divisions, (2) rectangular grid, (3) height and depth, (4) angles, (5) distances and (6) curves and lines.

[6] E.J. Dijksterhuis, De mechanisering van het Wereldbeeld. (The mechanistic View of the Universe). Meulenhoff, Amsterdam, 1950, pp. 213-221 (in Dutch).
The graphic display of physical phenomena in terms of longitudes and latitudes is described by Nicole Oresme in his 'Tractatus de Configurationibus Intensionum' and 'Tractatus de Latitudinibus Formarum'. Nicole Oresme was a French nominalist philosopher. He is also considered as the first monetary theorist. See also the chapter on Descartes and Oresme in this book.

[7] The significance of Nicole Oresme is discussed in:

Ernst Borchert, Die Lehre von der Bewegung bei Nikolas Oresme. Beiträge zur Geschichte der Philosophie und Theologie des Mittelalters, Band XXXI, Heft 3, Münster, 1934.
Anneliese Maier, Die Vorläufer Galileis im 14. Jahrhundert. Studien zur Naturphilosophie der Spätscholastik, Storia e Letteratura, Roma, 1949.
From a curvilinear velocity-time chart, Oresme derived the average uniform velocity which allowed an object to travel the same distance in the same time. This way, he may have discovered a form of integral calculus, 350 years before Newton and Leibniz, by using graphical means.

[8] Edward Wright, Certaine Errors in Navigation, arising either of the ordinarie erroneous making or using of the Sea Chart, Compasse, Cross Staffe, and Tables of Declinations of the Sunne, and fixed Starres detected and corrected. Valentine Sims, London, 1599. With an additional account of the voyage of the Earl of Cumberland to the Azores (1589) and a map of part of Europe and Africa in the exact Mercator projection.
Second revised and expanded edition in 1610. Facsimile edition by W.J. Johnson, Norwood, N.J., 1974.

[9] Arno Peters map, French edition by Akademische Verlagsanstalt FL-9490 Vadüz, Lichtenstein. Dutch edition by Wereldsolidariteit, Brussel, Belgium.
The elliptic Mollweide projection also conserves areas on the globe. The grid in this projection uses curved meridians and straight parallels.

[10] F. Van Cleemput, Mercator's projection. Mededelingen van de Marine Academie, Boek XVI, De Sikkel, Antwerpen, 1964, pp. 1-28 (in Dutch).
Nicholas
Crane, Mercator: The man who mapped the planet, Werdenfeld & Nicolson, London, 2002.

[11] D. Gernez, Quel procédé Mercator employa-t-il pour tracer le canevas de sa carte de 1569 à l'usage des marins? Mededelingen van de Akademie der Marine van België, Boek 1, 1936-37, pp. 146-171.

[12] Herman Nauts, Over de Mercator projectie op de wereldkaart (1569) (About the Mercator projection on the world globe of 1569). Special edition from the 'Oudheidkundige kring van het land van Waas', Nr. 15, St. Niklaas, 1962, pp. 245-262 (in Dutch).
Theoretical distances of the parallels from the equator have been compared with those measured on the original Mercator map by:
Emm. de Martonne, Traité de Géographie Physique. Paris, 1948.
A more detailed mathematical study of Mercator's projection has been performed by:
B. Kyewski, Ueber die Mercator-Projektion. Erdkunde, Laufg. 2, Band V, 1951.

[13] A detailed account of the contributions of Edward Wright is provided by:
Parsons E.J.S. and Morris W.F., Edward Wright and his Work. In: Imago Mundi, 3, 61-71, 1939. A periodical review of early cartography (Bagrow L. and Lynam E., Eds.) Reprint edition by N. Israel, Amsterdam, 1970.

[14] More recent biographic data on Edward Wright, including data from church records, is obtained from:
Wallis P.J., Edward Wright. In: Dictionary of Scientific Biography, Ch. C. Gillespie (Ed.) Vol. XIV, Ch. Scribner's Sons, New York, 1976, pp. 513-515.

[15] Anthony W.F. Edwards, Vanishing point. New Scientist, 128, 62-63, 1990.
Edwards was a fellow at Gonville and Caius in Cambridge where Wright had worked on his 'Certaine Errors in Navigation'.

[16] Parsons E.J.S. and Morris W.F., Edward Wright and his Work. Opus cit., p. 65.

[17] Edward Wright, Certaine Errors in Navigation. Opus cit.

[18] John Napier, Mirifici Logarithmorum Canonis Descriptio. 1614.
Wright's translation into English was published posthumously by his son Samuel in 1616 under the title 'Description of the admirable Table of Logarithms'.

[19] Wallis P.J., Edward Wright. Opus cit., p. 514. Quoted from D.W. Waters, The Art and Navigation in Elizabethan and Early Stuart Times. London, 1958.
The statement is similar to that which appears on the house in London at Warrington Crescent, that has been inhabited by another great English mathematician, Alan Mathieson Turing (1912-1954), who helped break the German Enigma code: 'Few individuals of whatever rank contributed more to the allied victory of World War Two'.

[20] Denis Wood, The power of maps. Scientific American, May 1993, pp. 88-93.
'This is the contradiction maps present: a claim to represent objectively a world they can only subjectively present, a claim to win acceptance for a view of the world whose utility lies precisely in its partiality.'

[21] Bruno Latour, in: Representation in Scientific Practice, M. Lynch and S. Woolgar (Eds.), MIT Press, Cambridge, MA, 1991. Book review by Ian Hacking, Matters of Graphics. Science, 252, 979-980, 1991.

Bruno Latour, Pandora’s hope: essays on the reality of science studies, Harvard Univ. Press, Cambridge, MASS, 1999.

Historical Notes on early Mapmaking

6th BC Pythagoras proposes the spherical shape of the earth.

4th BC Aristotle defines climatic parallels (climata).

3rd BC Eratosthenes, geographer and astronomer at Alexandria, defines meridians through well-known places (Alexandria, Rhodes,...). The perimeter of the earth is determined at 40,000 km from gnomonic measurements of the height of the sun.

1st BC Posidonius erroneously determines the length of the equator at 29,000 km from measurements of the declinations of the star Canopus. The rectangular grid is used in Greece for mapmaking.

87-150 AD Claudius Ptolemaeus (Ptolemy), geographer, astronomer and theorist of music at Alexandria, compiles the Cosmographia (also referred to as Geographia), which contains a world map and 26 local maps on a conical projection.

267 Phei Hsui (or Pei-Xui) proposes the six mapmaking rules in China and recommends rectangular gradual scales.

until 5th cent. Rectangular coordinates are used by Romans for cadastral mapping. Decline of cartography. Mapmaking is restricted to itineraries.

Middle-Ages Portolan or coastal maps for navigation. T-O world maps with religious character.

1397 Manuel Chrisoloras brings Ptolemaeus's Cosmographia from Byzantium to Florence.

1406 Ptolemaeus's Cosmographia is translated in Latin by Jacopo Angelo, and renamed Cosmographia.

1474 Johann Müller (Regiomontanus) prints maps and astronomical tables, which were possibly used by Columbus on his voyage of 1492.

1511 First documented map of Europe on a rectangular grid with increasing distances between parallels, produced by Erhardt Etzlaub on the back of a sundial for the use by pilgrims to Rome.

1524 Sebastian Münster prints the Cosmographia Universalis based on Ptolemaeus's work.

1533 Peter Bienewitz (Apianus) prints the Cosmographia, with an extension on triangulation by Gemma Frisius.

˜1550 Flemish engravers reach a high level of proficiency and compete with traditional German wood cutters.

1569 Gerard Mercator engraves a world map on a rectangular grid with increasing distances between parallels.

1570 Abraham Ortelius publishes the Theatrum Orbis Terrarum.

1578 Gerard de Jode produces the Speculum Orbis Terrarum.

1595 Posthumous edition of Mercator's Atlas by his son Rumoldus.

1599 Edward Wright describes the form of the Mercator projection in 'Certaine Errors in Navigation'.

1606 Jodocus Hondius published the Mercator-Hondius Atlas.

1635 Willem Blaeu and his son Jan produce a new Theatrum Orbis Terrarum.

1646 Robert Dudley compiles Dell Arcano del Mare, an Atlas of sea charts.

1656 Christian Huyghens invents the pendulum clock for the measurement of longitudes.

1669 Jean Dominique Cassini computes tables of eclipses of the moons of Jupiter for the measurement of longitude.

1699 The sextant is invented by Isaac Newton for the measurement of latitudes with a precision of less than 10 minutes (10 miles) depending upon weather conditions.

1736 John Harrison invents a chronometer for the determination of longitudes at sea and wins a prize of 20,000 pounds offered by the Board of Longitude in 1707. The instrument kept time within 15 seconds during 165 consecutive days.

18th cent. French cartographers design modern maps.

Biographical Notes on Mercator

1512	Born as Ge(e)rard De Cremer in Rupelmonde.
1532	Studies mathematics under the guidance of Gemma Frisius in Leuven and obtains the degree of Magister Artium.
1534	Learns engraving and instrument making (probably in Antwerp).
1537	Opens a workshop in Leuven, producing globes, instruments and maps. Map of Palestine (Terra Sancta).
1540	Map of Flanders, based on triangulations by Gemma Frisius. Treatise on calligraphy (Literarum Latinarum).
1541	Earth globe with loxodromes, constructed for Nicolas de Granvelle, chancellor of emperor Charles V.
1544	Suspected of 'Lutheranism', incarcerated during three or four months at the castle of Rupelmonde.
1551	Heaven globe constructed for prince-bishop George of Austria.
1552	Leaves for Duisburg in Germany, where he establishes a workshop.
1554	Map of Europe.
1560	Appointed surveyor and cartographer of the duke of Gullik and Kleef (Cleves) at Duisburg.
1569	World map using the method of increasing distances between parallels 'ad usum navigantium' .
1578	Edition of Ptolemaeus's Cosmographia.
1585	Series of 51 maps of France, Switzerland, Germany and the Netherlands.
1589	Series of 23 maps of Italy and the Balkan countries.
1594	G. Mercator dies at Duisburg.
1595	Posthumous edition of the Atlas by his son Rumoldus (Atlas sive cosmographicae Meditationes de Fabrica Mundi et Fabricati Figura).

1602 Jodocus Hondius acquires the Mercator plates after the death of Rumoldus.

1606 The Mercator-Hondius Atlas appears in Amsterdam.

Biographical Notes on E. Wright

1561	Born at Garveston, near Norfolk.
1576	Enters Gonville and Caius College in Cambridge.
1581-84	Possibly absent from Cambridge and employed at sea.
1587	Obtains a fellowship at Gonville and Caius.
1589	On leave by royal permission, joins the expedition of Georges, Earl of Cumberland, to the Azores, to prey upon and acquire booty from Spanish ships.
1592	Presents the manuscript of 'Certaine Errors in Navigation' to the Earl of Cumberland. The book provides a mathematical justification of Mercator's projection, a discussion of the variation of the compass and of the errors arising from the use of the cross staff and various astronomical tables.
1594	Moves from Cambridge to London. Thomas Blundeville publishes Wright's table of the subdivision of the meridian according to the Mercator projection in his 'Exercises containing Sixe Treatises' with due acknowledgement.
1596/97	Jodocus Hondius publishes the 'Christian Knight' world maps upon the Mercator projection as formulated by Wright, but without giving credit to the author.
1597	William Barlow publishes Wright's table in 'The Navigator's Supply', only with cryptic reference to the author.
1598	Publication of two world charts in the new projection by Richard Hakluyt in his 'Principal Navigations'.
1598/99	A navigator (presumably Abraham Kendall) who accompanied Francis Drake on his last expedition, intends to publish Wright's manuscript in his own name, but dies during the voyage.
1599	First edition of 'Certaine Errors'. Translation of Simon Stevin's 'Havenvinding' (The Havenfinding Art).
1600	Assists William Gilbert in his compilation of 'De Magnete' and contributes a preface. The book proposes a theory of the earth's magnetism and discusses the variation of the compass.
1608/09	Appointed tutor to prince Henry.

1612 Appointed lecturer in navigation in London, with support from the East-India Company. Surveyor for the New River project for bringing water to London.

1614 Translation of John Napier's work on logarithms 'Mirifici Logarithmorum Canonis Descriptio' into English. The work is edited posthumously by Henry Briggs and published by Wright's son Samuel in 1616 as 'Description of the admirable Table of Logarithms'.

1615 Edward Wright dies in London.